COMPUTATION OF THERMAL RESISTANCE OF LOW-TEMPERATURE HEAT TUBES

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Computation is carried out of thermal resistance in low-temperature tubes with the effect of the interrelation between evaporator and condenser on thermal resistance taken into account.

High values of heat fluxes and relatively small temperature drops are a characteristic feature of heat transmission in heat tubes. However, for low-temperature heat tubes with liquids and cores of low coefficients of thermal conductivity, the temperature difference between the outer surfaces of the evaporator and of the condenser may prove to be considerable and should be taken into account in the design of systems using heat tubes.

If, following [1], one assumes that condensation takes place directly on the inner surface of the core and one neglects the thermal resistance of the phase change and of the walls of the heat tube, one can obtain in the one-dimensional case the following equation which governs the heat and mass transfer in the condenser:

$$\frac{d^2\theta_c}{d\xi^2} + A_c \xi \frac{d\theta_c}{d\xi} = 0, \tag{1}$$

where

$$\theta_c = \frac{T_c}{T_{sat}},\ \xi = \frac{y}{\delta},\ A_c = \frac{jc_p\delta}{bL_c\lambda_{ef}},$$

and the Pomerants criterion is

$$\mathrm{Po}_c = \frac{Q\delta}{bL_c\lambda_{ef}T_{sat}}.$$

If the boundary conditions

$$\frac{d\theta_c}{d\xi} = \mathrm{Po}_c \text{ for } \xi = 0 \text{ and } \theta_c = 1 \text{ for } \xi = 1 \tag{2}$$

are satisfied, then the solution of Eq. (1) is given by

$$\theta_c = 1 - \mathrm{Po}_c \sqrt{\frac{2\pi}{A_c}}\,[\Phi^{-1}(\sqrt{A_c}) - \Phi^{-1}(\xi\sqrt{A_c})]. \tag{3}$$

In the above the error integral is given by

$$\Phi^{-1}(x) = \frac{1}{\sqrt{2\pi}}\int_0^x \exp\left(-\frac{t^2}{2}\right)dt.$$

For the outer surface of the condenser the value of the temperature is as follows:

$$\theta_{c.o} = 1 - \mathrm{Po}_c \sqrt{\frac{2\pi}{A_c}}\,\Phi^{-1}(\sqrt{A_c}). \tag{4}$$

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The equation for the evaporator is obtained under the assumption that the heat emission by evaporation takes place from the surface of the core and that the temperature field in the liquid flow which flows from the condenser is given by the relation (3) and that it differs from the temperature distribution along the core thickness of the evaporator. This results in the inhomogeneous equation

$$-\frac{d^2\theta_e}{d\xi^2}+A_e\xi\frac{d\theta_e}{d\xi}+A_e\theta_e=A_e\theta_c, \tag{5}$$

whose solution is sought for the following boundary conditions:

$$-\frac{d\theta}{d\xi}=\mathrm{Po_e}\ \text{ for }\ \xi=0,\ \theta_e=1\ \text{ for }\ \xi=1. \tag{6}$$

In the above

$$\theta=\frac{T_e}{T_{sat}},\quad A_e=\frac{jc_p\delta}{bL_e\lambda_{ef}},\quad \mathrm{Po_e}=\frac{Q\delta}{bL_e\lambda_{ef}T_{sat}}.$$

A particular solution of the homogeneous equation associated with the inhomogeneous one in (5) is

$$\varphi=\exp\left(\frac{A_e\xi^2}{2}\right). \tag{7}$$

Following [2] the general solution of Eq. (5) is obtained by

$$\theta_e=C_1\varphi+C_2\varphi\int\frac{d\xi}{E\varphi^2}+\varphi\int\frac{1}{E\varphi^2}\left(\int E\varphi n d\xi\right)d\xi, \tag{8}$$

in which one has for the problem under consideration

$$E=\exp\left(-\frac{A_e\xi^2}{2}\right),\quad n=-A_e\theta_c.$$

The constants C_1 and C_2 are given as follows:

$$C_1=\exp\left(\frac{A_e}{2}\right)\ \text{ for }\ \xi=1,\ C_2=-\mathrm{Po_e}\ \text{ for }\ \xi=0.$$

By using the expressions (3) for θ_c the temperature in the evaporator core is found from (8) and is of the form

$$\theta_e=\theta_c+\mathrm{Po_e}\frac{\sqrt{2\pi(A_c+A_e)}}{A_e}\exp\left(\frac{A_e\xi^2}{2}\right)[\Phi^{-1}(\sqrt{A_c+A_e})-\Phi^{-1}(\xi;\ \overline{A_c+A_e})]. \tag{9}$$

Then the temperature of the outer surface of the evaporator is

$$\theta_{e.o}=\theta_{c.o}+\mathrm{Po_e}\frac{\sqrt{2\pi(A_c+A_e)}}{A_e}\Phi^{-1}(\sqrt{A_c+A_e}). \tag{10}$$

The temperature drop between the outer surfaces of the evaporator and the condenser is obtained from (10) and (4), namely,

$$\Delta\theta_e=\mathrm{Po_e}\frac{\sqrt{2\pi(A_c+A_e)}}{A_e}\Phi^{-1}(\sqrt{A_c+A_e}). \tag{11}$$

It can be seen from (11) that the temperature drop can only be evaluated if the fluid flow found in the expressions for A_c and A_e is known in the core j and differs from its highest possible value by the transportation properties of the core and of the fluid. The value of j is found from the heat-balance equation for the condenser

$$j(r+h_{sat})=Q+j\bar{h}. \tag{12}$$

In the above $\bar{h}$ denotes the mean integral value of the liquid enthalpy over the core thickness,

$$\bar{h}=\frac{1}{\delta}\int_0^\delta[h_{sat}+c_p(T_c-T_{sat})]\,dy. \tag{13}$$

After reorganizing by employing (3) and (13), as well as the expression for A_c, one obtains a transcendental equation which determines uniquely the value of the liquid flow, namely,

$$j = \frac{Q}{r} \exp\left(-\frac{jc_p\delta}{2bL_c\lambda_{ef}}\right). \tag{14}$$

The transformations have been carried out under the assumption that the evaporator directly adjoins the condenser. In the case in which the evaporator and the condenser are separated by an adiabatic zone it is advisable when forming the equations for the evaporator that the temperature of the liquid flowing into the evaporator be considered as equal to the mean integral value over the core thickness in the condenser. In such case the computational relations can be obtained in the same way as above.

NOTATION

h_{sat}, liquid enthalpy at saturation temperature, J/kg; T_e, T_c, temperatures in the cores of the evaporator and condenser, °K; $T_{e.o}$, $T_{c.o}$, temperatures of the outer surfaces of the evaporator and condenser, °K; b, δ, width and thickness of the core, m; L_e, L_c, evaporator and condenser lengths, respectively, m; c_p, heat capacity of liquid, J/kg·°K; λ_{ef}, effective thermal conductivity of core filled with liquid, W/m·°K; j, liquid flow in the core, kg/sec; r, latent heat of vaporization, J/kg; Q, loading of thermal tube, W.

LITERATURE CITED

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